Temperature Compensation of Coaxial Cavities*

J. R. COGDELL†, A. P. DEAM‡ AND A. W. STRAITON‡

Summary—**This paper describes a technique for temperature compensation of coaxial cavities by controlling the capacitance between the end of the center conductor and an end plate across the outer conductor. A formula is derived for this capacitance which is verified experimentally. Supplemental design data are also obtained experimentally.**

Introduction

A DROP-TYPE atmospheric refractometer operating at 403 mc has recently been reported by A. P. Deam.[1,2] The sensing element for this unit is a coaxial cavity whose resonant frequency controls a Pound stabilized oscillator.[3] In this cavity application a cavity which is tunable in the normal sense, over a wide bandwidth, is not necessary. Instead, the cavity is effectively tuned by the atmospheric dielectric constant which is purposely vented into the cavity. Montgomery[4] has described the construction and compensation of coaxial cavities for tuning over a relatively large bandwidth. In the case of the frequency standard cavity, as the desired cavity might be considered, simpler bimetallic techniques may be employed to effectively utilize the capacitance discontinuity for compensation.

In the design process, leading to cavity development, calculation of the absolute resonant frequency was desired with as great an accuracy as possible since no manual tuning would be allowed. Approximations[5] to the discontinuity at the open end of a $\lambda/4$ cavity have been derived. However, none of these yields the accuracy desired, and an effort was made to represent the discontinuity more accurately. This discontinuity expression is included as part of the cavity design.

The cavity chosen to be compensated is designated by Ramo and Whinnery[6] as a "foreshortened coaxial line." The cavity, as shown in Fig. 1, is very similar to an ordinary quarter wavelength coaxial cavity and differs only in that its outer conductor is extended past the inner conductor for a short distance and closed with a conducting plane. When the gap between the center conductor and the endplate is small compared with a wavelength, it acts as a simple capacitance. The equivalent circuit which approximately represents the electrical properties of the cavity is that of a transmission line of length L_p terminated with a capacitance. This circuit is shown in Fig. 2. The resonant frequency of the cavity is a function of the length of the center conductor and the length of the end gap. Compensation requires that thermally-caused changes in the dimensions of the center conductor and gap produce opposing changes of equal magnitude in the cavity's frequency.

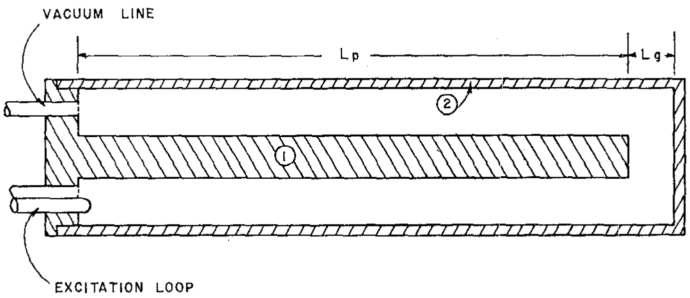

Fig. 1—Section of temperature-compensated cavity.

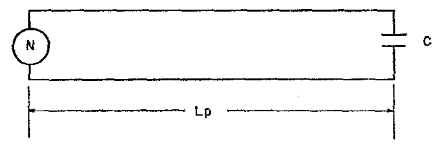

Fig. 2—Equivalent circuit of type II cavity.

Design

When the cavity is resonant, in any plane, susceptances looking in opposite directions will be equal in magnitude and opposite in sign. At the plane of the end of the center conductor, the susceptance looking into the line will be that of a shorted transmission line of length L_p, and the susceptance looking into the gap will be that of a simple capacitance. Equating these susceptances gives

$$Y_0 \operatorname{ctn} \frac{2\pi L_p f_r}{c} = 2\pi f_r C \tag{1}$$

where

C = capacitance of end space in farads,
Y_0 = characteristic admittance of coaxial line in mhos,
c = velocity of light.

* Manuscript received by the PGMTT, July 27, 1959; revised manuscript received, October 5, 1959.

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[1] A. P. Deam, "An Expendable Atmospheric Radio Refractometer," Elec. Engrg. Res. Lab., Rept. No. 108; University of Texas, Austin, Tex.; May, 1959.

[2] A. P. Deam, "An Expendable Atmospheric Refractometer," presented at URSI Convention, Washington, D. C.; May, 1959.

[3] R. V. Pound, "Frequency stabilization of microwave oscillators," Proc. IRE, vol. 35, pp. 1405–1415; December, 1947.

[4] C. G. Montgomery, "Techniques of Microwave Measurements," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 11; 1947.

[5] N. Marcuvitz, "Waveguide Handbook," M.I.T. Rad. Lab. Ser., vol. 10; 1951.

[6] S. Ramo and J. R. Whinnery, "Waves and Fields in Modern Radio," John Wiley and Sons, Inc., New York, N. Y., p. 415; 1953.

The end capacitance is kept small in these cavities and does not have a great effect on the frequency of the cavity. The length of the center conductor is, therefore, nearly one-fourth of a wavelength, and the following is a good approximation:

$$\operatorname{ctn}\frac{2\pi L_p f_r}{c} = \frac{\pi}{2} - \frac{2L_p f_r}{c}\,. \tag{2}$$

Substitution of (2) into (1) and rearrangement produces

$$f_r = \frac{c}{4\left[L_p + \dfrac{cC}{Y_0}\right]}\,. \tag{3}$$

The capacitance of the end configuration as shown in Fig. 3 will now be considered. A potential difference between the plates is assumed, the fields in the enclosed space are determined, the surface charges are calculated, and the capacitance is evaluated by the familiar formula, $C = q/V_0$.

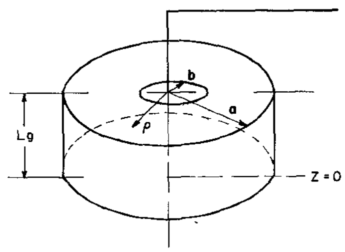

Fig. 3—End capacitance configuration.

A solution to Laplace's equation suitable for the boundary conditions is

$$V = \Sigma(A_n \cosh \lambda_n z + B_n \sinh \lambda_n z)J_0(\lambda_n \rho), \tag{4}$$

where A_n, B_n, and λ_n, the separation constant, may take on any value. The boundary conditions are

$$z = 0 \qquad V = 0 \tag{5}$$

$$\rho = a \qquad V = 0 \tag{6}$$

$$\begin{aligned} z = L_g \qquad V &= V_0 \qquad && 0 < \rho < b \\ &= V_0 \frac{\ln a/\rho}{\ln a/b} \qquad && b < \rho < a. \end{aligned} \tag{7}$$

A logarithmic potential distribution was chosen for (7) because this is the normal distribution in a coaxial line. Boundary conditions (5) and (6) can be satisfied if

$$A_n = 0, \qquad J_0(a\lambda_n) = 0,$$

and boundary condition (7) can be satisfied if $B_n \sinh \lambda_n L_g$ are made the Fourier coefficients of an expansion of (7) in Bessel functions.

$$B_n \sinh \lambda_n L_g = \frac{2}{a^2 J_1^2(\lambda_n a)}\left[\int_0^b V_0 \rho J_0(\lambda_n \rho) d\rho + \int_b^a \frac{V_0 l_n a/\rho}{l_n a/b} \rho J_0(\lambda_n \rho)\right], \tag{8}$$

$$B_n = \frac{2V_0 J_0(\lambda_n b)}{a^2 l_n a/b \lambda_n^2 \sinh \lambda_n L_g J_1^2(\lambda_n a)}\,. \tag{8a}$$

The potential in the space is therefore

$$V = \sum_{n=1}^{\infty} \frac{2V_0 J_0(\lambda_n b) \sinh \lambda_n z J_0(\lambda_n \rho)}{a^2 l_n a/b \lambda_n^2 J_1^2(\lambda_n b) \sinh \lambda_n L_g}\,. \tag{9}$$

The known potential defines the charge density on the surfaces, and the total charge on either plate can be found readily. The capacitance can be found by dividing the total charge on either plate by the total voltage, V_0. In an ordinary capacitor, the total charge on both surfaces will be equal. This is not necessarily true in this case, since a potential distribution was assumed at the plane of $z = L_g$. The total charges, in fact, are different and lead to different values for the capacitance. Considering the charges on the smaller surface, the capacitance is calculated to be

$$C = \sum_{n=1}^{\infty} \frac{4\pi\epsilon b J_0(\lambda_n b) J_1(\lambda_n b) \coth \lambda_n L_g}{a^2 l_n a/b \lambda_n^2 J_1^2(\lambda_n a)}, \tag{10}$$

and the capacitance following from the charge on the larger surface is

$$C = \sum_{n=1}^{\infty} \frac{4\pi\epsilon a J_0(\lambda_n b) J_1(\lambda_n b) \coth \lambda_n L_g}{a^2 l_n a/b \lambda_n^2 J_1^2(\lambda_n a)}\,. \tag{11}$$

The two values differ by a factor of a/b with (11)giving the larger value. It might be expected that the proper value for the capacitance lies somewhere between those given by (10) and (11). Let the proper value be expressed by

$$C = \sum_{n=1}^{\infty} \frac{4\pi\epsilon\delta J_0(\lambda_n b) J_1(\lambda_n b) \coth \lambda_n L_g}{a^2 l_n a/b \lambda_n^2 J_1^2(\lambda_n a)} \tag{12}$$

where δ is an effective radius. The correct value for δ can be found by referring to (3) which shows that the cavity will have the same resonant frequency as an open circuited line of length $L_p + cC/Y$. A coaxial line which is terminated in a circular waveguide operating beyond cutoff will have an effective length, $L_p + d$. The apparent lengthening of the center conductor, given by the term d, expresses the effect of the discontinuity which is produced at the plane where the coaxial line is terminated in the circular waveguide.

As the gap length, L_g, is increased, the term cC/Y_0 should approach the value of d produced by the discontinuity mentioned. This value of d may be determined from formulas or tables and used to evaluate δ.

As an example, consider an application to a specific cavity with inner and outer radii of 0.250 inch and 0.84 inch, respectively. Previous experience of this laboratory shows that d is approximately 0.290 inch for cavities of this type. This value fixes the value of δ at 0.651 inch and gives a value of capacitance which is between those of (10) and (11). With these values, the resonant frequency of the cavity under consideration is

$$f_r = \frac{c}{4\left[L_p + \dfrac{4\pi\epsilon(0.651)c}{Y_0(0.80)^2 l_n \dfrac{0.84}{0.25}} \displaystyle\sum_{n=1}^{\infty} \frac{J_0(0.250\lambda_n)J_1(0.250\lambda_n)}{\lambda_n^2 J_1^2(0.840\lambda_n)}\right]}, \qquad (13)$$

which reduces to

$$f_r = \frac{c}{4L_p + \dfrac{2(0.651)}{(0.84)^2} \displaystyle\sum_{n=1}^{\infty} \frac{J_0(0.250\lambda_n)J_1(0.250\lambda_n)}{\lambda_n^2 J_1^2(0.840\lambda_n)}}. \qquad (13a)$$

Temperature compensation will now be considered. The rate of change of resonant frequency, f_r, with temperature θ, is given by

$$\frac{df_r}{d\theta} = \frac{\partial f_r}{\partial L_g}\frac{dL_g}{d\theta} + \frac{\partial f_r}{\partial L_p}\frac{dL_p}{d\theta} = 0. \qquad (14)$$

The solution to (14) is of interest in the region of the resonant frequency, 403 mc, and the value of L_p (and therefore of $\partial f_r/\partial L_p$) does not vary widely even when L_g is varied over a wide range. This justifies the fact that a constant, evaluated for (13a), can be used for $\partial f_r/\partial L_p$ in (14).

The derivatives of the two dimensions with respect to temperature can be evaluated easily enough. If the expansion coefficients of the inner and outer conductors are denoted by α_1 and α_2, respectively, then

$$\frac{dL_p}{d\theta} = \alpha_1 L_p \qquad (15)$$

and

$$\frac{dL_g}{d\theta} = \alpha_2(L_p + L_g) - \alpha_1 L_p. \qquad (16)$$

Substitution of these into (14) and rearrangement produces

$$\frac{\partial f_r}{\partial L_g} = \frac{-\alpha_1 L_p}{L_p(\alpha_2 - \alpha_1) + L_g(\alpha_2)}\frac{\partial f_r}{\partial L_p}. \qquad (17)$$

Eq. (17) can be solved graphically for L_g by evaluating $\partial f_r/\partial L_g$ as a function of L_g from (13a), and plotting both sides of (17) against L_g. As a further special case, let the inner conductor be made of invar ($\alpha_1 \approx 1\times10^{-6}/C°$), the outer conductor be made of cold-rolled steel ($\alpha_2 \approx 12\times10^{-6}/C°$), and L_p be slightly less than a quarter wavelength, 7.05 inches, for instance. The solution of (17), using these values is shown in Fig. 4, and the correct value for L_g can be seen to be 0.63 inch. This dimension, when substituted into (13a), shows that $L_p = 7.025$ inches will give the correct frequency of 403 mc.

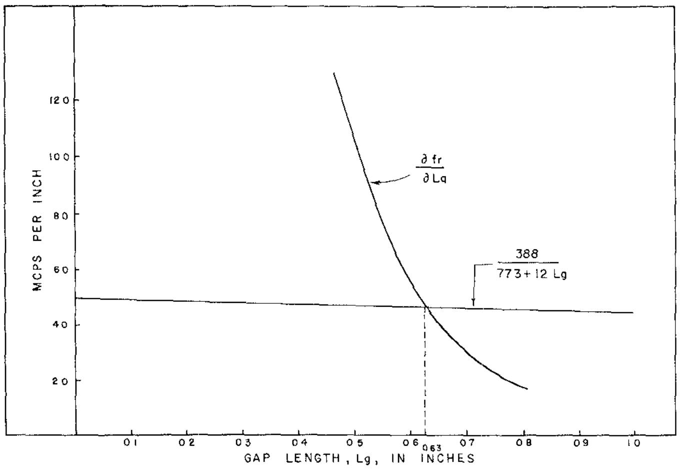

Fig. 4—Graphical solution of (9).

Experimental Work

Eq. (13a) was verified by making a cavity with a movable end-plate as shown in Fig. 5, and varying L_g while noting its effects on the resonant frequency. A Pound stabilized oscillator was coupled to the cavity by a loop which was adjusted for critical coupling, and the frequency of the oscillator was monitored with a Gertsch FM-3 frequency meter as the gap distance was varied. Fig. 6 shows the result of such a test. Although the theoretical curve gives slightly lower values (of the order of 0.2 per cent) than the experimental curve, the shapes of the curves are similar; thus, the dimensions calculated from (13a) should produce compensation since only the slope of the curve is used in that calculation.

Two aspects of the cavity which could not be approached analytically were investigated with the cavity which was constructed to verify (13). These were 1) the effect of venting the endplate, and 2) the effect of boring holes in the center conductor. A cavity used to sense the atmospheric refractive index might have both these features. The results of such tests are shown in Figs. 7 and 8. From these it can be seen that the shape of the fre-

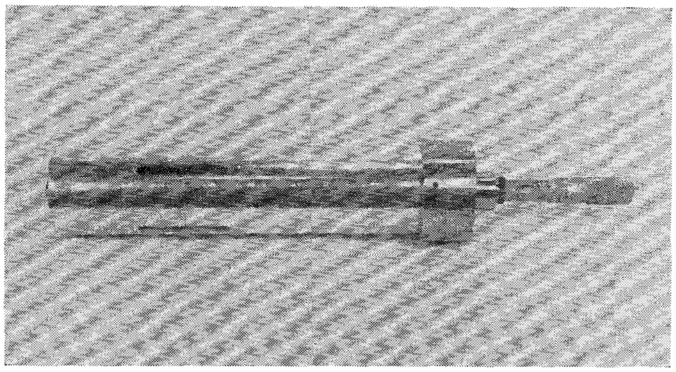

Fig. 5—Cavity with variable endplate in place.

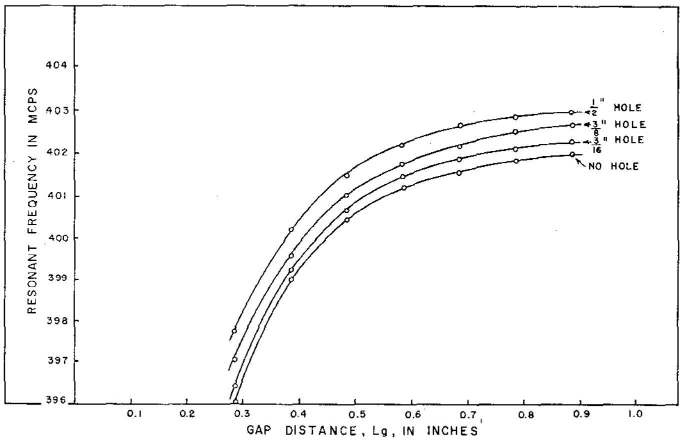

Fig. 8—Resonant frequency vs gap distance showing effect of boring holes in center conductor.

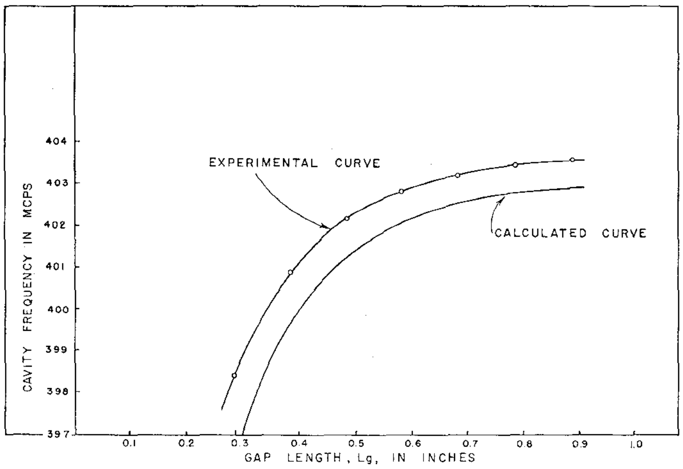

Fig. 6—Resonant frequency vs gap distance.

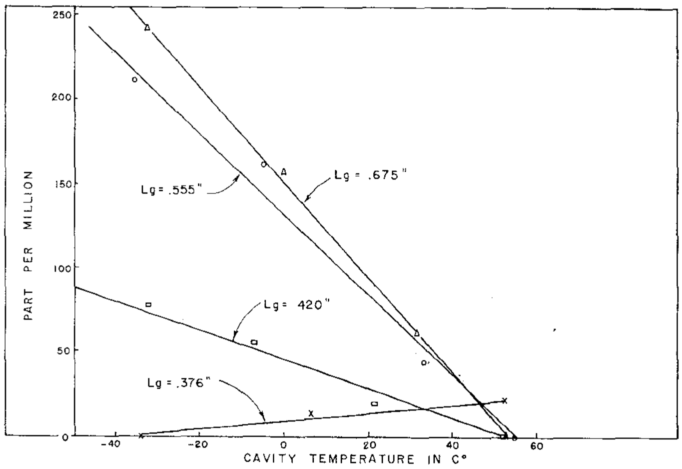

Fig. 9—Cavity compensation with different size gap lengths.

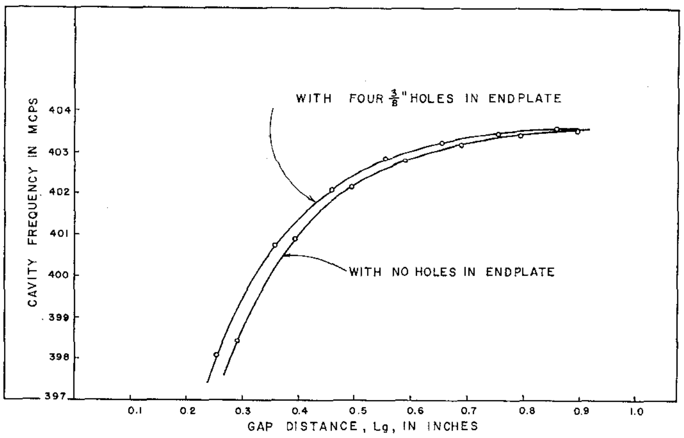

Fig. 7—Resonant frequency vs gap distance showing effect of venting endplate.

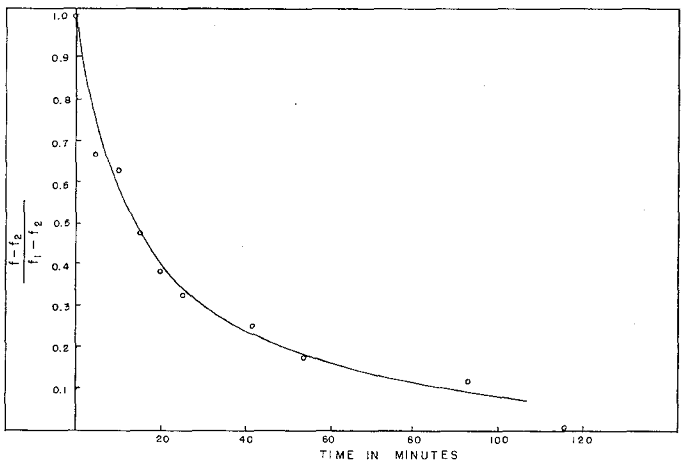

Fig. 10—Long-time transient response.

quency vs gap-distance curve is not affected greatly by boring holes in the endplate and center conductor; thus, the compensation would not be affected markedly by these modifications.

Results

Temperature Compensation

A cavity was constructed with the following dimensions: $b=0.250$, $a=0.840$, $L_g=0.675$, and $L_p=7.025$ (all dimensions in inches). The cavity had a resonant frequency of 403.5 mc, 0.2 per cent higher than (13a) would indicate, and an unloaded Q of 2280.

The temperature compensation of the cavity was measured by placing it in a constant-temperature bath and measuring its resonant frequency as a function of temperature. The cavity temperature was varied from $-32°$C to 53°C, and the temperature coefficient of the cavity was found to be 2.62 parts/million/C° undercompensation; *i.e.*, the cavity frequency was lowered as the temperature was raised. This was a rather surprising result since an uncompensated invar cavity would be expected to exhibit a temperature coefficient of 1×10^{-6}/C° undercompensation. The high coefficient indicated one of two things: either 1) the invar center conductor had a higher expansion coefficient than advertised, or 2) some unexpected effect (such as eccentricity of the center conductor, for example) was present. Whatever the cause of the undercompensation, the gap length had to be shortened to compensate for it. This was done in steps until a final gap length of 0.376 inch produced a temperature coefficient of 0.25×10^{-6}/C° overcompensation, which was considered satisfactory. Fig. 9 shows the effects of the various gap lengths on the compensation of the cavity.

Transient Response

During the temperature tests some data on the transient response of the cavity were obtained. The cavity experienced a rise in temperature when it was inserted into the temperature bath. The response of the cavity showed two time constants, one associated with the transfer of heat to the outer conductor and the other associated with the transfer of heat to the center conductor. These time constants were measured to be 1.1 minute and 29 minutes, respectively. These results agree fairly well with the response determined analytically. As shown in Fig. 10, the cavity reached thermal equilibrium in about 2 hours after it experienced a change in temperature.

It should be mentioned that these figures for the transient response are not those which would be experienced in actual use. When employed in atmospheric soundings, the rate of transfer of heat from the air to the cavity would be very slow so that it would be expected that the cavity would have essentially the same temperature throughout the center post and the body at a given time. Hence, the temperature compensation techniques described in this paper would be effective.

A Graphical Method for Measuring Dielectric Constants at Microwave Frequencies*

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Summary—**This paper describes a graphical method for measuring the real and imaginary parts of the dielectric constant $\epsilon/\epsilon_0=\epsilon'-j\epsilon''$ of materials at microwave frequencies. The method is based on the network approach to dielectric measurements proposed by Oliner and Altschuler in which the dielectric sample fills a section of transmission line or waveguide. In contrast to their method, the network representing the dielectric sample is analyzed in terms of the bilinear transformation**

$$\Gamma'=\frac{a\Gamma+b}{c\Gamma+d};\qquad ad-bc=4.$$

The analysis proceeds from the geometric properties of the image circle in the Γ plane obtained by terminating the output line in a calibrated sliding short.

The technique described retains the desirable features of the network approach but avoids the necessity of measuring both scattering coefficients. As a result the procedure is more direct and, in the case of the TEM configuration, leads to an entirely graphical solution in which the complex dielectric constant can be read from a Smith chart overlay.

* Manuscript received by the PGMTT, August 7, 1959; revised manuscript received October 9, 1959. This work was sponsored by the U. S. Army Signal Res. and Dev. Lab., Fort Monmouth, N. J.
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Introduction

There are many techniques for making dielectric measurements at microwave frequencies.[1] One of the more interesting methods proposed in recent years is that due to Oliner and Altschuler,[2] in which the

[1] A. von Hippel, ed., "Dielectric Materials and Applications," J. Wiley and Sons, Inc., New York, N. Y., ch. 2; 1954.

[2] A. Oliner and H. Altschuler, "Methods of measuring dielectric constants based upon a microwave network viewpoint," *J. Appl. Phys.*, vol. 26, pp. 214–219; February, 1955.